

17

Using a graphic display calculator

CHAPTER OBJECTIVES:

This chapter shows you how to use your graphic display calculator (GDC) to solve the different types of problems that you will meet in your course. You should not work through the whole of the chapter – it is simply here for reference purposes. When you are working on problems in the mathematical chapters, you can refer to this chapter for extra help with your GDC if you need it.

Instructions for the Casio fx-9860GII calculator

Use this list to help you find the topic you need

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Before you start

You should be familiar with:

- Important keys on the keyboard: AC/ON SHIFT DEL MENU X,θ,T EXE OPTN EXIT
- The home screen
- Using the Main Menu to enter the mode you want
- Changing window settings in GRAPH mode
- Using zoom tools in GRAPH mode
- Using trace in GRAPH mode

For a reminder of how to perform the basic operations have a look at your GDC manual.

1 Functions

1.1 Graphing linear functions

Example 1

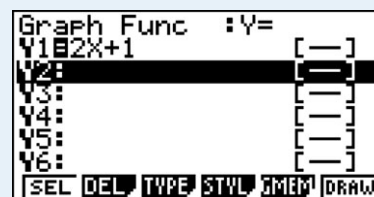
Draw the graph of the function $y = 2x + 1$.

Press **menu**. You will see the dialog box as shown on the right.
Choose 5: GRAPH and press **EXE**.

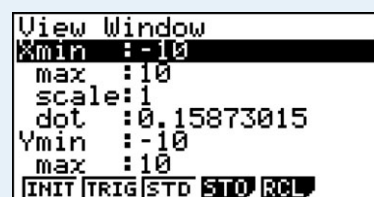


The default graph type is Function, so the form $Y=$ is displayed.
Type $2x + 1$ and press **EXE**.

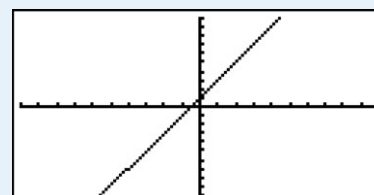
Use **X,θ,T** to enter x .



Press **SHIFT** **F3** V-Window and choose **F3** STD to use the default axes which are $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$.
Press **EXE** and **F6** DRAW.



The graph of $y = 2x + 1$ is now displayed on the screen.



Finding information about the graph

The GDC can give you a lot of information about the graph of a function, such as the coordinates of points of interest and the gradient (slope).

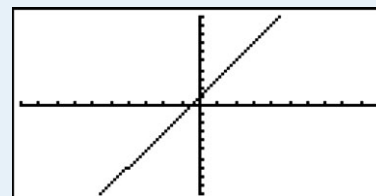
1.2 Finding a root

The x -intercept is known as a *root* of the function.

Example 2

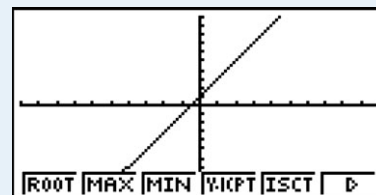
Find the root of $y = 2x + 1$.

Draw the graph of $y = 2x + 1$ as in Example 15.

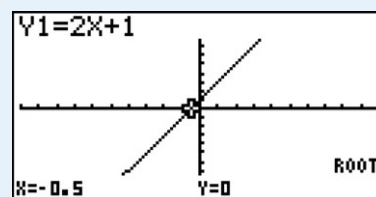


Press **SHIFT** **F5** G-Solv.

Press **F1** Root.



The calculator displays the root of the function $y = 2x + 1$ at the point $(-0.5, 0)$.



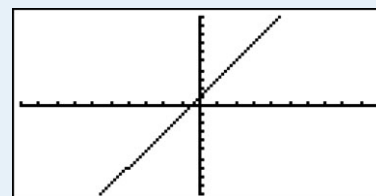
1.3 Finding the gradient (slope) of a line

The correct mathematical notation for gradient (slope) is $\frac{dy}{dx}$. You will find out more about this in the chapter on differential calculus. Here we just need to know this is the notation that will give us the gradient (slope) of the line.

Example 3

Find the gradient of $y = 2x + 1$.

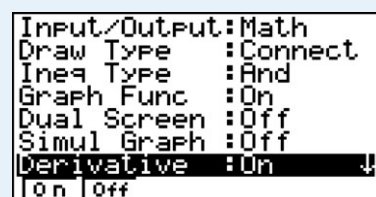
First draw the graph of $y = 2x + 1$ (see Example 15).



Press **SHIFT** (SET UP).

Set Derivative to On.

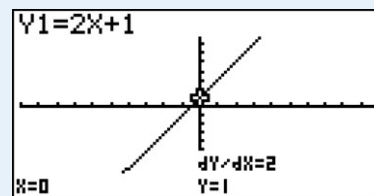
Press **EXE** and **F6** DRAW to return to the graph.



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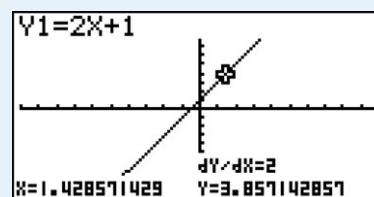
Press **SHIFT** **F1** Trace.

The calculator displays the coordinates of the point and the gradient.



Move the point along the line using the **◀** and **▶** keys.

The gradient (slope) is shown by dy/dx and is 2 at every point along the line.



1.4 Solving simultaneous equations graphically

To solve simultaneous equations graphically you draw the straight lines and then find their point of intersection. The coordinates of the point of intersection give you the solutions x and y .

Note: The calculator will only draw the graphs of functions that are expressed explicitly. By that we mean functions that begin with 'y =' and have a function that involves only x to the right of the equals sign. If the equations are written in a different form, you will need to rearrange them before using your calculator to solve them.

Solving simultaneous equations using a non-graphical method is covered in section 1.5.

Example 4

Solve the simultaneous equations $2x + y = 10$ and $x - y = 2$ graphically with your GDC.

First rearrange both equations in the form $y =$

$$\begin{array}{rcl} 2x + y = 10 & & x - y = 2 \\ y = 10 - 2x & & -y = 2 - x \\ & & y = x - 2 \end{array}$$

To draw graphs $y = 10 - 2x$ and $y = x - 2$:

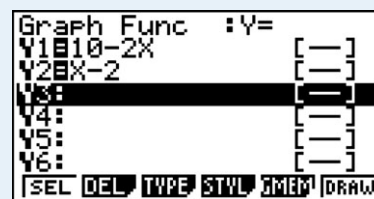
Press **menu** and choose 5: GRAPH and press **EXE**.

The default graph type is Function, so the form $Y=$ is displayed.

Type $10 - 2x$ and press **EXE** and $x - 2$ and press **EXE**.

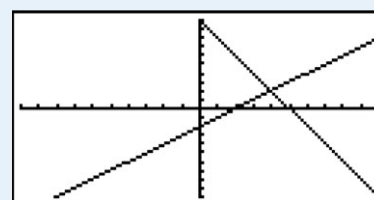
Press **SHIFT** **F3** V-Window and choose **F3** STD to use the default axes which are $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$.

Press **EXE** and **F6** DRAW



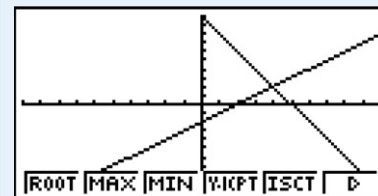
If the calculator displays a graph, press **EXIT** to return to this screen.

The calculator displays both straight line graphs
 $Y1 = 10 - 2x$ and $Y2 = x - 2$.



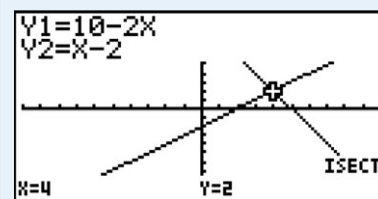
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Press **F5** G-Solv and **F5** ISCT.



The calculator displays the intersection of the two straight lines at the point(4, 2).

The solutions are $x = 4$, $y = 2$.



Simultaneous and quadratic equations

1.5 Solving simultaneous linear equations

When solving simultaneous equations in an examination, you do not need to show any method of solution. You should simply write out the equations in the correct form and then give the solutions. The calculator will do all the working for you.

Example 5

Solve the equations:

$$2x + y = 10$$

$$x - y = 2$$

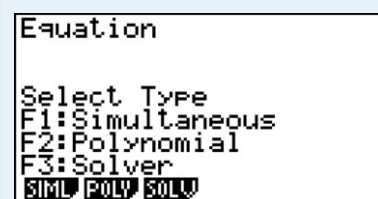
Press **menu**. You will see the dialog box as shown on the right.

Choose A: EQUA and press **EXE**.



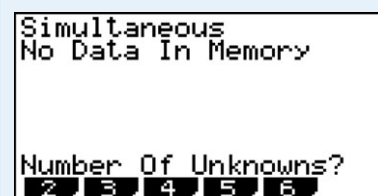
From the menu, choose Simultaneous and press **F1**.

If there are previous equations in the memory, press **EXIT** until you return to this menu.



Choose equations in two unknowns and press **F1**.

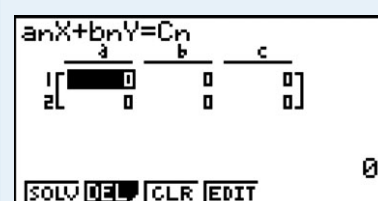
Note: This is how you will use the linear equation solver in your examinations. In your project, you might want to solve a more complicated system with more equations and more variables.



You will see the template on the right.

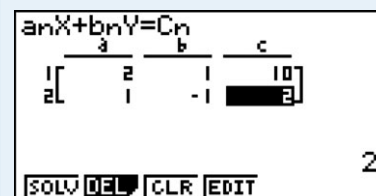
Type the coefficients from two equations into the template, pressing **EXE** after each number.

The equations must be in the correct order.

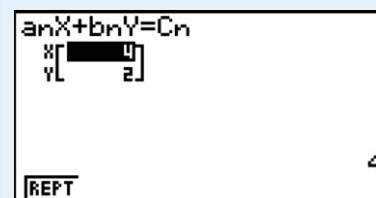


► Continued on next page

Press **F1** and the calculator will solve the equations, giving the solutions as X and Y.



The solutions are $x = 4$, $y = 2$.



Quadratic functions

1.6 Drawing a quadratic graph

Example 6

Draw the graph of $y = x^2 - 2x + 3$ and display it using suitable axes.

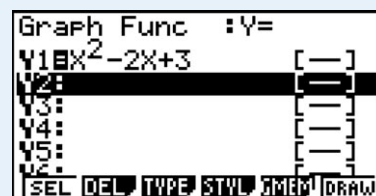
Press **menu** and choose 5: GRAPH and press **EXE**.

The default graph type is Function, so the form $Y=$ is displayed.

Type $y = x^2 - 2x + 3$ and press **EXE**.

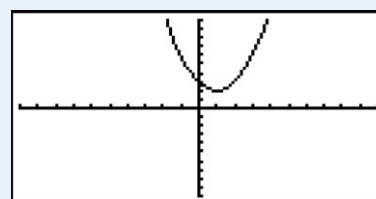
Press **SHIFT** **F3** V-Window and choose **F3** STD to use the default axes which are $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$.

Press **EXE** and **F6** DRAW.

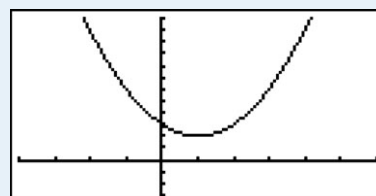


If the calculator displays a graph, press **EXIT** to return to this screen.

The calculator displays the curve with the default axes.



Adjust the window to make the quadratic curve fit the screen better.



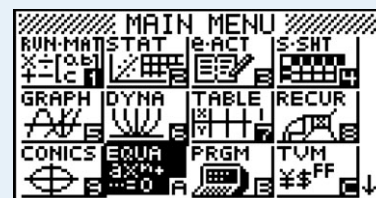
1.7 Solving quadratic equations

When solving quadratic equations in an examination, you do not need to show any method of solution. You should simply write out the equations in the correct form and then give the solutions. The GDC will do all the working for you.

Example 7

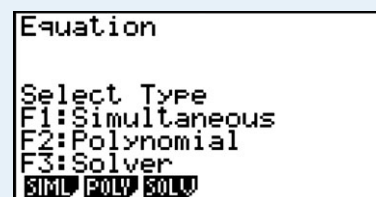
Solve $3x^2 - 4x - 2 = 0$

Press **menu**. You will see the dialog box as shown on the right.
Choose A: EQUA and press **EXE**.



From the menu, choose Polynomial and press **F2**.

If there are previous equations in the memory, press **EXIT** until you return to this menu.



You will see the dialog box as shown on the right. Choose equations of Degree2 and press **F1**.



Another dialog box opens for you to enter the equation.

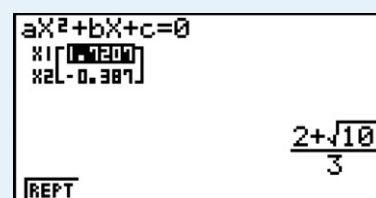
The general form of the quadratic equation is $aX^2 + bX + c = 0$ so we enter the coefficients in a , b and c .



Here $a = 3$, $b = -4$ and $c = -2$. Be sure to use the **(-)** key to enter the negative values. Press **EXE** after each value.
Press **F1** and the calculator will find the roots of the equation.



The solutions are $x = -0.387$ or $x = 1.72$ (3 sf).

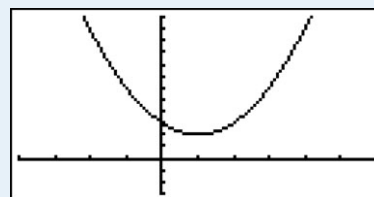


1.8 Finding a local minimum or maximum point

Example 8

Find the minimum point on the graph of $y = x^2 - 2x + 3$.

Draw the graph of $y = x^2 - 2x + 3$ (See Example 19).



Method 1 - using a table

You can look at a table of the values of the function.

Press **menu** and choose 7: TABLE and press **EXE**.



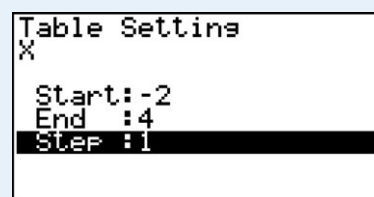
Press **F5** SET.



Choose a start and end point for the table and a step of 1.

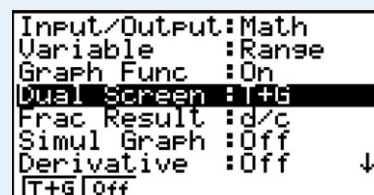
Use the x -values from the graph you drew.

Press **EXIT**.



Press **SHIFT** SET UP.

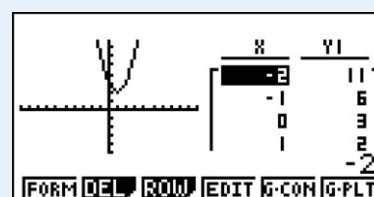
Scroll down to Dual Screen and press **F1** T+G.



Press **EXE** **F6** to view the table.

X	Y1
-2	11
-1	6
0	3
1	2
2	3

Press **F5** G·CON to display the graph alongside the table.

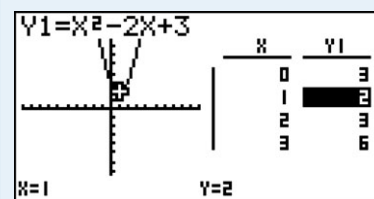


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Press **OPTN** **F2** GLINK.

When you scroll through the table the cross moves along the curve.

The minimum value shown in the table is 2 when $x = 1$.



Look more closely at the values of the function around $x = 1$.

Press **SHIFT** **F3** V-Window.

Choose values $-0.9 \leq x \leq 1.1$ and $1.99 \leq y \leq 2.01$ and press **EXE**.

Change the settings in the table: Press **F1** **F5** SET.

Set Start to 0.9

End to 1.1

Step to 0.01

Press **EXE** **F6** to view the graph and the table.

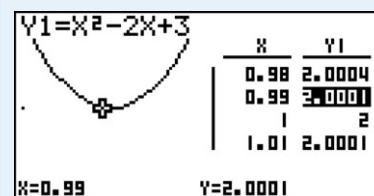


Press **F5** G-CON to display the graph alongside the table.

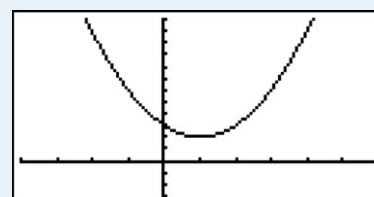
Press **OPTN** **F2** GLINK and scroll through the table.

The table shows that the function has larger values at points around (1, 2).

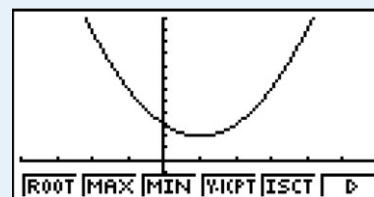
We can conclude that this is a local minimum on the curve.



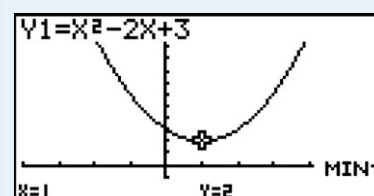
Method 2 - Using the minimum function



Press **F5** G-Solv **F3** MIN.



The calculator displays the minimum point on the curve at (1, 2).



Example 9

Find the maximum point on the graph of $y = -x^2 + 3x - 4$.

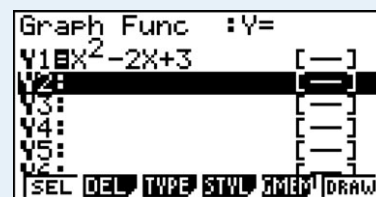
Press **menu** and choose 5: GRAPH and press **EXE**.

The default graph type is Function, so the form $Y =$ is displayed.

Type $-x^2 + 3x - 4$ and press **EXE**.

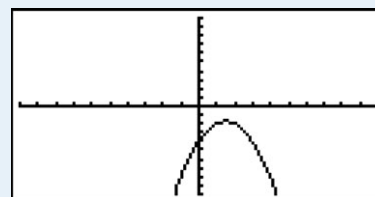
Press **SHIFT** **F3** V-Window and choose **F3** STD to use the default axes which are $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$.

Press **EXE** and **F6** DRAW.

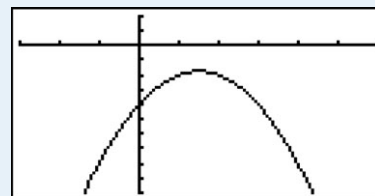


If the calculator displays a graph, press **EXIT** to return to this screen.

The calculator displays the curve with the default axes.



Adjust the window to make the quadratic curve fit the screen better.



Method 1 - using a table

You can look at a table of the values of the function.

Press **menu** and choose 7: TABLE and press **EXE**.



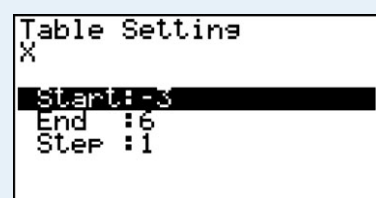
Press **F5** SET.



Choose a start and end point for the table and a step of 1.

Use the x -values from the graph you drew.

Press **EXIT**.



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Press **SHIFT** **SET UP**.
 Scroll down to Dual Screen and press **F1** T+G.

```

Input/Output:Math
Variable      :Range
Graph Func    :On
Dual Screen   :T+G
Frac Result   :d/c
Simul Graph   :Off
Derivative    :Off
T+G           :Off
  
```

Press **EXE** **F6** to view the table.

X	Y1
-3	-22
-2	-14
-1	-8
0	-4
1	-2
2	-4

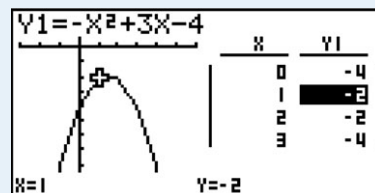
FORM DEL ROW EDIT G·CON G·PLT

Press **F5** G·CON to display the graph alongside the table.



Press **OPTN** **F2** GLINK.

When you scroll through the table the cross moves along the curve.
 The minimum value shown in the table is -2 when $x = 1$ and $x = 2$.



Look more closely at the values of the function around $x = 1$ and $x = 2$.

Press **SHIFT** **F3** V-Window.

Choose values $1 \leq x \leq 2$ and $-2.5 \leq y \leq -1.5$ and press **EXE**.

Change the settings in the table: Press **F1** **F5** SET.

Set Start to 0.9

End to 1.1

Step to 0.01

Press **EXE** **F6** to view the graph and the table.

```

View Window
Xmin :1
max :2
scale:1
dot :0.01612903
Ymin :-2.5
max :-1.5
INIT TRIG STD STO RCL
  
```

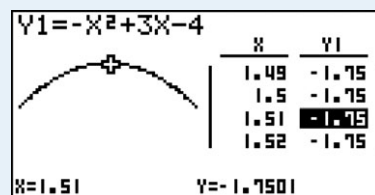
```

Table Settings
X
Start:0.9
End :1.1
Step :0.01
  
```

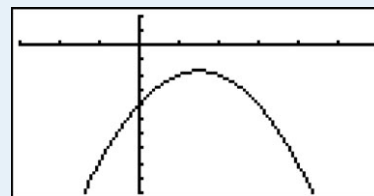
Press **F5** G·CON to display the graph alongside the table.

Press **OPTN** **F2** GLINK and scroll through the table.

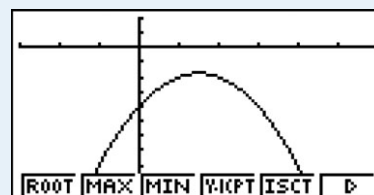
The table shows that the function has larger values at points around $(1.5, -1.75)$. We can conclude that this is a local maximum on the curve.



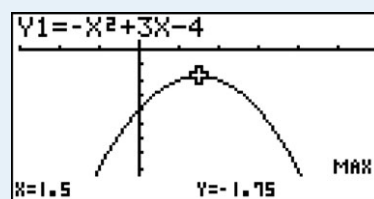
▶ Continued on next page

Method 2 - Using the maximum function

Press **F5** G-Solv **F3** MIN.



The calculator displays the maximum point on the curve at (1.5, -1.75).



Exponential functions

1.9 Drawing an exponential graph

Example 10

Draw the graph of $y = 3^x + 2$.

Press **menu** and choose 5: GRAPH and press **EXE**.

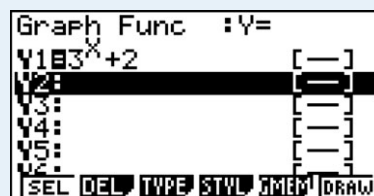
The default graph type is Function, so the form $Y=$ is displayed.

Type $3^x + 2$ and press **EXE**.

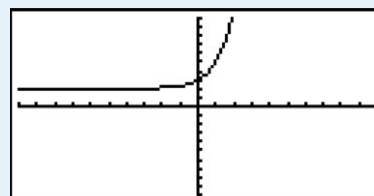
Press **SHIFT** **F3** V-Window and choose **F3** STD to use the default axes which are $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$.

Press **EXE** and **F6** DRAW.

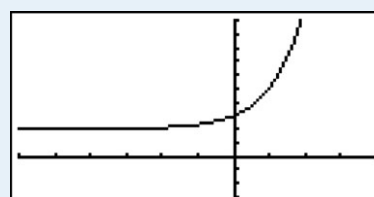
Note: Type **3** **^** **X,θ,T** **▶** to enter 3^x .
The **▶** returns you to the baseline from the exponent.



The calculator displays the curve with the default axes.



Adjust the window to make the exponential curve fit the screen better.

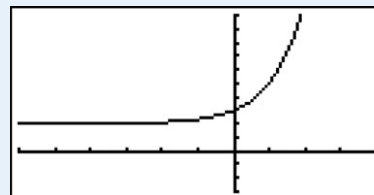


1.10 Finding a horizontal asymptote

Example 11

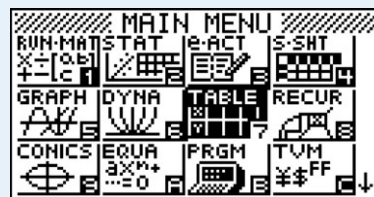
Find a horizontal asymptote to the graph of $y = 3^x + 2$.

Draw the graph of $y = 3^x + 2$ (See Example 22).



You can look at a table of the values of the function.

Press **menu** and choose 7: TABLE and press **EXE**.



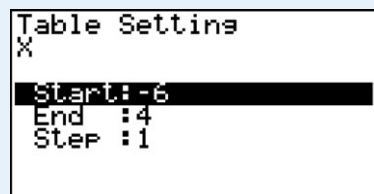
Press **F5** SET



Choose a start and end point for the table and a step of 1.

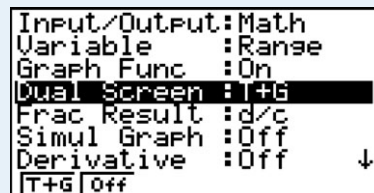
Use the x -values from the graph you drew.

Press **EXIT**.



Press **SHIFT** SET UP.

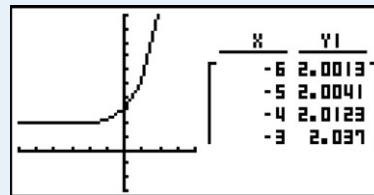
Scroll down to Dual Screen and press **F1** T+G.



Press **EXE** **F6** to view the table.

X	Y1
-6	2.0013
-5	2.0041
-4	2.0123
-3	2.0371
-2	2.0946
-1	2.2512
0	2.5
1	3.5
2	5.5
3	11.5
4	23.5
5	47.5
6	95.5

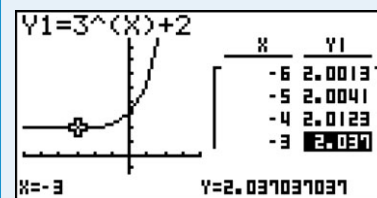
Press **F5** G·CON to display the graph alongside the table.



▶ Continued on next page

Press **OPTN** **F2** GLINK.

As the value of x gets smaller, Y_1 gets closer and closer to 2.



Press **EXIT** and then press **F5** SET.

Change the minimum value of x to -12 .

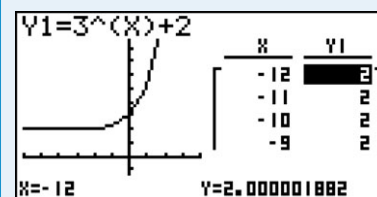
EXE **F6** **F5** G·CON **OPTN** **F2** GLINK.

Eventually the value of Y_1 displayed in the table reaches 2.

You can see, at the bottom of the screen, that the actual value of Y_1 is 2.000001882...

We can say that $Y_1 \rightarrow 2$ as $x \rightarrow -\infty$.

The line $x = 2$ is a horizontal asymptote to the curve $y = 3^x + 2$.



Logarithmic functions

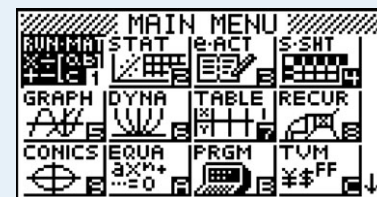
1.11 Evaluating logarithms

Example 12

Evaluate $\log_{10} 3.95$, $\ln 10.2$ and $\log_5 2$.

Press **menu**. You will see the dialog box as shown on the right.

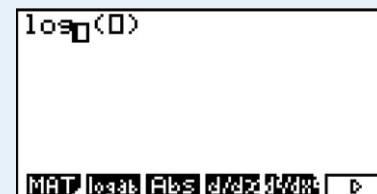
Choose 1: RUN·MAT and press **EXE**.



Press **EXIT** until you see the menu on the right, then press **F4** MATH.

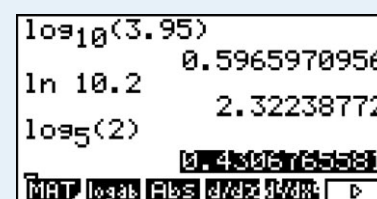


Press **F2** logab to open the logarithm template.



For natural logarithms it is possible to use the same method, with the base equal to e , but it is quicker to press **ln**. Similarly for logarithms to base 10 you can press **log**.

Note that the GDC will evaluate logarithms with any base without having to use the change of base formula.



1.12 Finding an inverse function

The inverse of a function can be found by interchanging the x and y values. Geometrically this can be done by reflecting points in the line $y = x$.

Example 13

Show that the inverse of the function $y = 10^x$ is $y = \log_{10} x$ by reflecting $y = 10^x$ in the line $y = x$.

Press **menu**. You will see the dialog box as shown on the right.



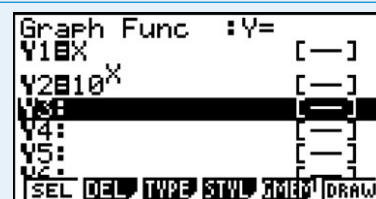
Draw the line $y = x$ so that it can be recognised as the axis of reflection. Choose 5: GRAPH and press **EXE**.

Type x and press **EXE**.

Type 10^x and press **EXE**.

Use **X,θ,T** to enter x .

Note: Type 10 **1** **0** **^** **X,θ,T** to enter 10^x .

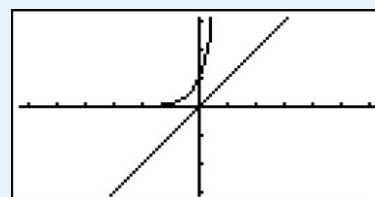


Press **SHIFT** **F3** V-Window and choose **F1** INIT to set up square axes which are $-6.3 \leq x \leq 6.3$ and $-3.1 \leq y \leq 3.1$.



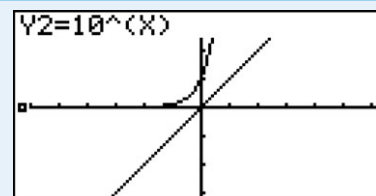
Press **EXE** and **F6** DRAW.

The graphs of $y = x$ and $y = 10^x$ are displayed.



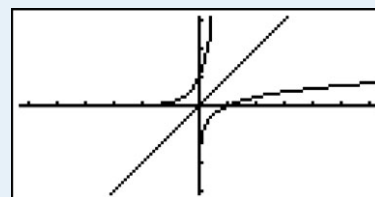
Press **SHIFT** **F4** Sketch | **F4** Inv.

Press **↑** to select the curve $Y2 = 10^x$.



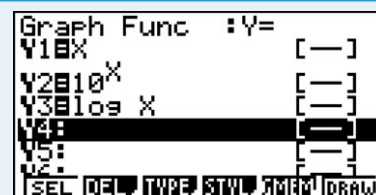
Press **EXE**.

The calculator will display the inverse of the function $y = 10^x$.



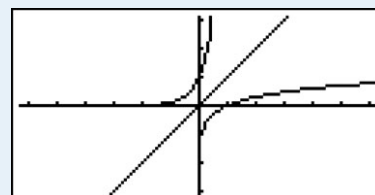
Press **EXIT** to display the Y= editor. Type $\log(x)$.

Press **log** **X,θ,T** to enter $\log(x)$. **log** is a shorter way to enter \log_{10} .



Press **EXE** and **F6** DRAW to display the graphs of $y = x$, $y = 10^x$ and $y = \log_{10} x$.

The inverse function and the logarithmic function coincide, showing that $y = \log_{10} x$ is the inverse of the function $y = 10^x$.



1.13 Drawing a logarithmic graph

Example 14

Draw the graph of $y = 2\log_{10}x + 3$.

Press **menu**. You will see the dialog box as shown on the right.



Choose 5: GRAPH and press **EXE**.

Type $2\log(x) + 3$ and press **EXE**.

Press **2** **log** **X,θ,T** **+** **3**

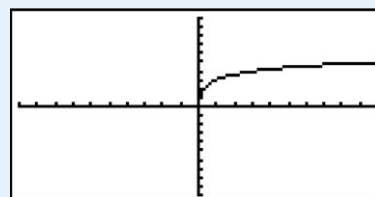


Press **SHIFT** **F3** V-Window and choose **F3** STD for the default axes which are $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$.

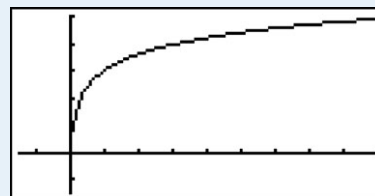


Press **EXE** and **F6** DRAW.

The graphs of $y = 10y = 2\log_{10}x + 3$ is displayed with the default axes.



Change the axes to make the logarithmic curve fit the screen better.



Trigonometric functions

1.14 Degrees and radians

Work in trigonometry will be carried out either in degrees or radians. It is important, therefore, to be able to check which mode the calculator is in and to be able to switch back and forth.

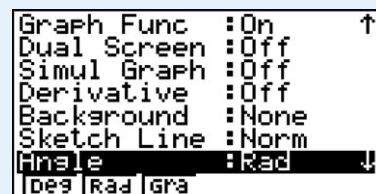
Example 15

Change angle settings from radians to degrees and from degrees to radians.

Press **SHIFT** **SET UP**.

Select either Deg or Rad using **F1** or **F2**.

Press **EXE**.



More complicated functions

1.16 Solving a combined quadratic and exponential equation

Follow the same GDC procedure when solving simultaneous equations graphically and solving a combined quadratic and exponential equation. See Examples 18 and 24.

To solve the equation, find the point of intersection between the quadratic function $y_1 = x^2 - 2x + 3$ and the exponential function $y_2 = 3 \times 2^{-x} + 3$.

Example 17

Solve the equation $x^2 - 2x + 3 = 3 \cdot 2^{-x} + 4$.

Press **menu** and choose 5: GRAPH and press **EXE**.

The default graph type is Function, so the form $Y=$ is displayed.

Type $x^2 - 2x + 3$ in Y_1 and press **EXE**.

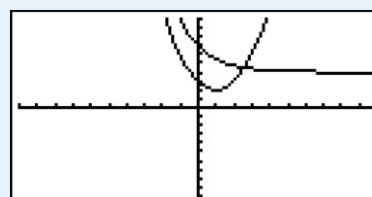
Then type $3 \times 2^{-x} + 4$ in Y_2 and press **EXE**.

Press **SHIFT** **F3** V-Window and choose **F3** STD to use the default axes which are $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$.

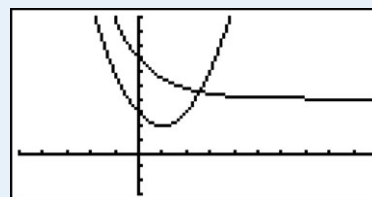
Press **EXE** and **F6** DRAW.

Note: Type **2** **^** **(-)** **X,T** **▶** to enter 2^{-x} .
The **▶** returns you to the baseline from the exponent.

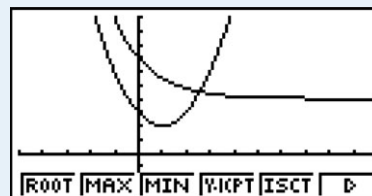
The calculator displays the curves with the default axes.



Adjust the window to make the quadratic curve fit the screen better.

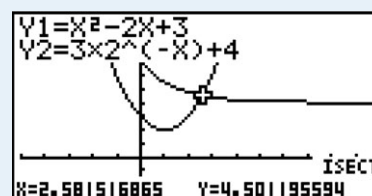


Press **F5** G-Solv and **F5** ISCT.



The calculator displays the intersection of the two straight lines at the point (2.58, 4.50).

The solutions are $x = 2.58$ and $y = 4.50$.



1.17 Using sinusoidal regression

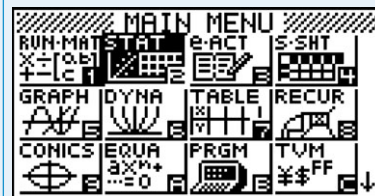
Example 18

It is known that the following data can be modelled using a sine curve.

x	0	1	2	3	4	5	6	7
y	6.9	9.4	7.9	6.7	9.2	8.3	6.5	8.9

Use sine regression to find a function to model this data.

Press **menu**. You will see the dialog box as shown on the right.
Choose the 2: STAT and press **EXE**.



Type the x -values in the first column (List 1) and the y -values in the second column (List 2).

Press **EXE** after each number to move down to the next cell. Press **▶** to move to the next column.

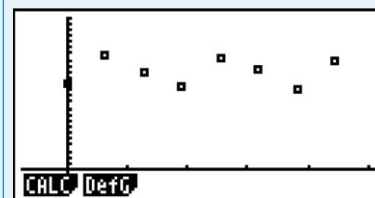
You can use columns from List 1 to List 26 to enter the lists.

	List 1	List 2	List 3	List 4
SUB				
1	0	6.9		
2	1	9.4		
3	2	7.9		
4	3	6.7		

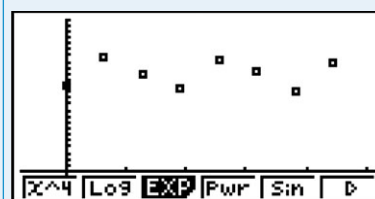
Press **F1** GRPH and **F6** SET.
Select Graph Type and choose **F1** Scatter.
Press **EXE**.
Press **F1** GPH1.



The automatic scales do not always give the best display of the box and scatter diagram. You cannot use V-Window to change the default values, but you can zoom in or out.
Press **F1** CALC.



Press **F6** **▶** and **F5** Sin.

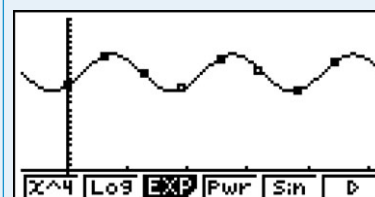


On screen, you will see the result of the sinusoidal regression.
The equation is in the form $y = a\sin(bx + c) + d$ and you will see the values of a , b , c and d displayed separately.

The equation of the sinusoidal regression line is
 $y = 1.51\sin(2.00x - 0.80) + 7.99$

```
SinReg
a =1.50600052
b =2.00290101
c =-0.7998736
d =7.99107865
MSe=3.0615e-04
y=a* sin(bx+c)+d
COPY DRAW
```

Press **F6** DRAW to return to the Graphs page.



2 Differential calculus

Finding gradients, tangents and maximum and minimum points

2.1 Finding the gradient at a point

Example 19

Find the gradient of the cubic function $y = x^3 - 2x^2 - 6x + 5$ at the point where $x = 1.5$.

Press **menu** and choose 5: GRAPH and press **EXE**.

The default graph type is Function, so the form $Y=$ is displayed.

Type $y = x^3 - 2x^2 - 6x + 5$ and press **EXE**.

Press **SHIFT** **F3** V-Window and choose **F3** STD to use the default axes which are $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$.

Press **EXE** and **F6** DRAW.

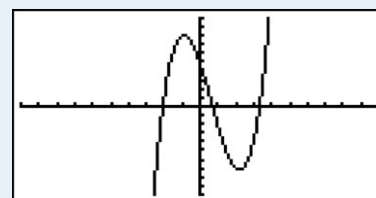
Note: Type **X,θ,T** **^**

3 **▶** to enter x^3 .

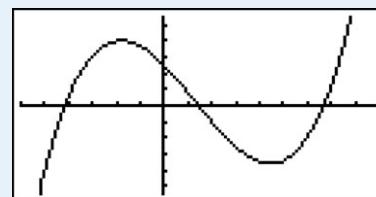
The **▶** returns you to the baseline from the exponent.

Graph Func :Y=
Y1=X³-2X²-6X+5
Y2:
Y3:
Y4:
Y5:
SEL DEL TYPE STYL ZMEM DRAW

The calculator displays the curve with the default axes.



Adjust the window to make the cubic curve fit the screen better.



Press **SHIFT** (SET UP).

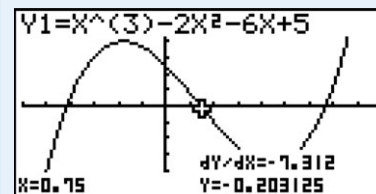
Set Derivative to On.

Press **EXE** and **F6** DRAW to return to the graph.

Input/Output:Math
Draw Type :Connect
Ineq Type :And
Graph Func :On
Dual Screen :Off
Simul Graph :Off
Derivative :On
On | Off

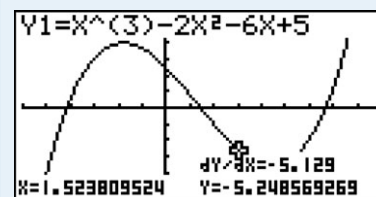
Press **SHIFT** **F1** Trace.

The calculator displays the coordinates of the point and the gradient.

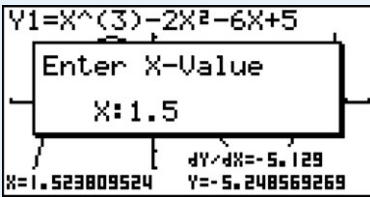
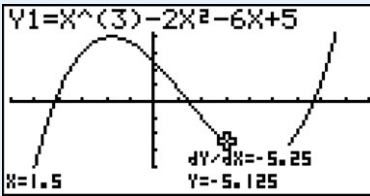


Move the point along the line using the **◀** and **▶** keys.

The gradient (slope) can be found at a point that is close to 1.5.

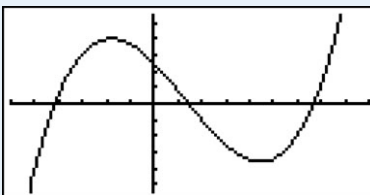
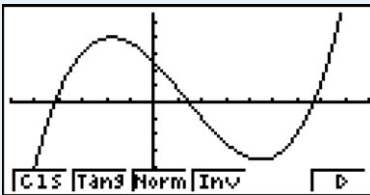
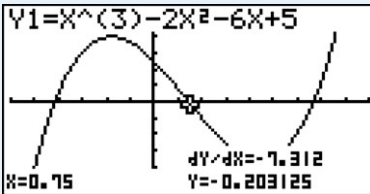
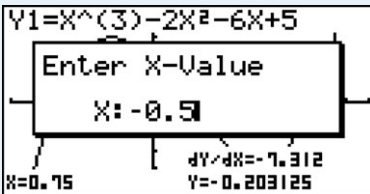
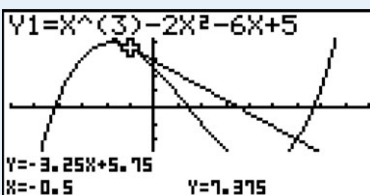


▶ Continued on next page

Type $\boxed{1} \boxed{.} \boxed{5} \boxed{\text{EXE}}$.	
The calculator displays the gradient of the curve at the point where $x = 1.5$. The gradient is -5.25 .	

2.2 Drawing a tangent to a curve

Example 20

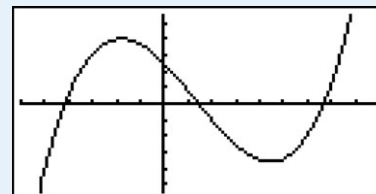
Draw a tangent to the curve $y = x^3 - 2x^2 - 6x + 5$ where $x = -0.5$.	
First draw the graph of $y = x^3 - 2x^2 - 6x + 5$ (see Example 30).	
Press $\boxed{\text{F4}}$ Sketch. Press $\boxed{\text{F2}}$ Tang.	
The calculator displays the trace screen.	
Press $\boxed{(-)} \boxed{0} \boxed{.} \boxed{5} \boxed{\text{EXE}}$. Press $\boxed{\text{EXE}}$.	
The equation of the tangent is $y = -3.25x + 5.75$.	

2.3 Finding maximum and minimum points

Example 21

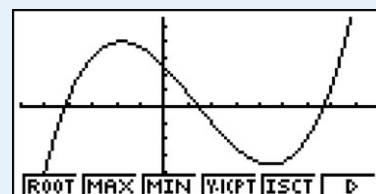
Find the local maximum and local minimum points on the cubic curve.

First draw the graph of $y = x^3 - 2x^2 - 6x + 5$ (see Example 30).

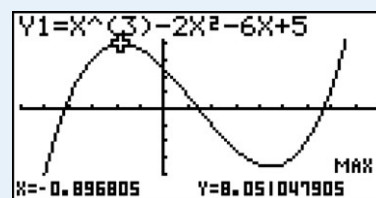


Press **SHIFT** **F5** G-Solv.

Press **F2** MAX to find the maximum point.



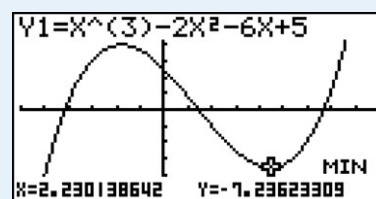
The calculator displays the local maximum at the point $(-0.897, 8.05)$.



Press **SHIFT** **F5** G-Solv.

Press **F3** MIN to find the minimum point.

The calculator displays the local minimum at the point $(2.23, -7.24)$.



2.4 Finding a numerical derivative

Using the calculator it is possible to find the numerical value of any derivative for any value of x .

The calculator will not, however, differentiate a function algebraically. This is equivalent to finding the gradient at a point graphically.

Example 22

If $y = \frac{x+3}{x}$, evaluate $\frac{dy}{dx} \Big|_{x=2}$

Press **MENU**. You will see the dialog box as shown on the right.

Choose 1: RUN·MAT and press **EXE**.

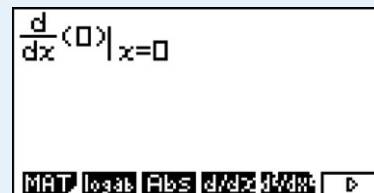


Press **EXIT** until you see the menu on the right, then press **F4** MATH.

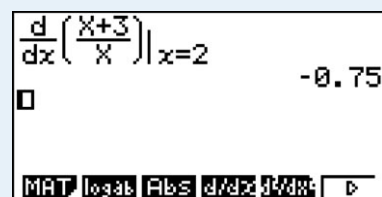


▶ Continued on next page

Press F4 $\frac{d}{dx}$ to open the differentiation template.



The calculator shows that the value of the first derivative of $y = \frac{x+3}{x}$ is -0.75 when $x = 2$.



2.5 Graphing a numerical derivative

Although the calculator can only evaluate a numerical derivative at a point, it will graph the gradient function for all values of x .

Example 23

If $y = \frac{x}{x+3}$, draw the graph of $\frac{dy}{dx}$.

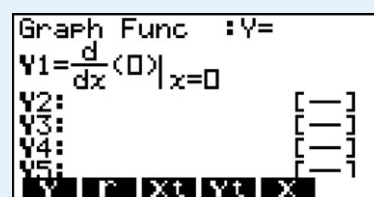
Press MENU . You will see the dialog box as shown on the right.



Choose 5: GRAPH and press EXE .
Press OPTN F2 CALC.



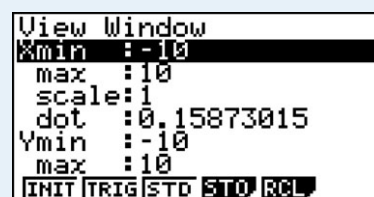
Choose F1 $\frac{d}{dx}$ to choose the derivative template.



In the template enter x , the function $\frac{x}{x+3}$ and the value x .
Press EXE .



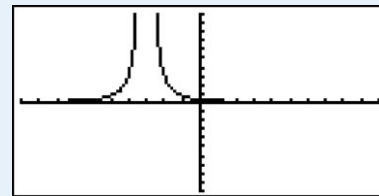
Press SHIFT F3 V-Window and choose F3 STD for the default axes which are $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$.



► Continued on next page

Press **EXE** and **F6** DRAW.

The GDC displays the graph of the numerical derivative function of $y = \frac{x}{x+3}$.



Example 24

Find the values of x on the curve $y = \frac{x^3}{3} + x^2 - 5x + 1$ where the gradient is 3.

Press **MENU**. You will see the dialog box as shown on the right.

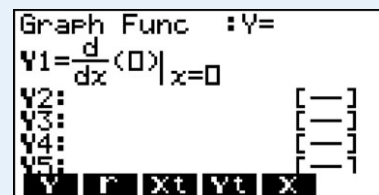


Choose 5: GRAPH and press **EXE**.

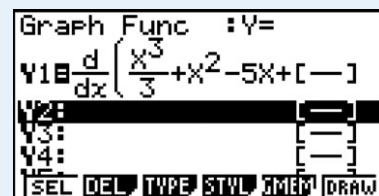
Press **OPTN** **F2** CALC.



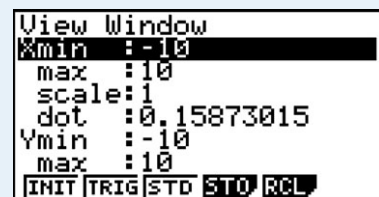
Choose **F1** $\frac{d}{dx}$ to choose the derivative template.



In the template enter x , the function $\frac{x^3}{3} + x^2 - 5x + 1$ and the value x . Press **EXE**.

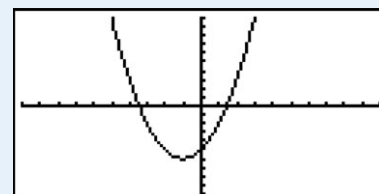


Press **SHIFT** **F3** V-Window and choose STD for the default axes which are $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$.



Press **EXE** and **F6** DRAW.

The calculator displays the graph of the numerical derivative function of $y = \frac{x^3}{3} + x^2 - 5x + 1$.

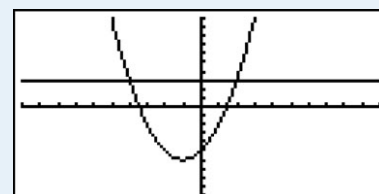


Press **EXIT** to display the Y= editor.

Enter the function $Y_2 = 3$.

Press **F6** DRAW.

The calculator now displays the curve and the line $y = 3$.



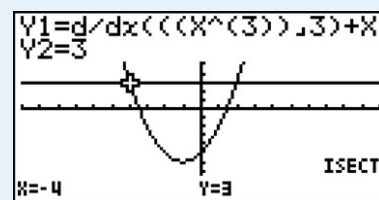
▶ Continued on next page

To find the points of intersection between the curve and the line:

Press **SHIFT** **F5** G-Solv.

Press **F5** ISCT.

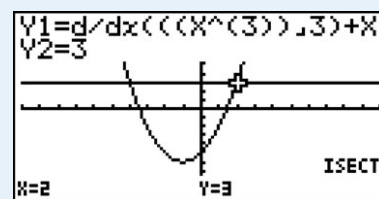
The GDC shows a point of intersection at (4, 3).



Press **▶** to select the second point.

The GDC shows a point of intersection at (2, 3).

The curve has gradient 3 when $x = -4$ and $x = 2$.



2.6 Using the second derivative

The calculator can find first and second derivatives. The second derivative can be used to determine whether a point is a maximum or minimum point.

Example 25

Find the stationary points on the curve $f(x) = x^4 - 4x^3$ and determine their nature.

$$f(x) = x^4 - 4x^3$$

$$f'(x) = 4x^3 - 12x^2$$

At stationary points

$$f'(x) = 0$$

$$4x^3 - 12x^2 = 0$$

$$4x^2(x - 3) = 0$$

Therefore $x = 0$ or $x = 3$.

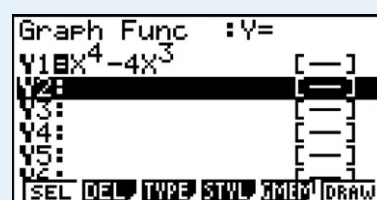
Press **MENU**. You will see the dialog box as shown on the right.



Choose 5: GRAPH and press **EXE**.



Enter $x^4 - 4x^3$ for Y1.



Press **MENU**. You will see the dialog box as shown on the right.

Choose 1: RUN·MAT and press **EXE**.

Use **▲** to enter the exponent.
Press **▶** to return to the base line.



▶ Continued on next page

Use the calculator to find the coordinates of the points and to determine their nature.

Evaluate the function when $x = 0$ and $x = 3$.

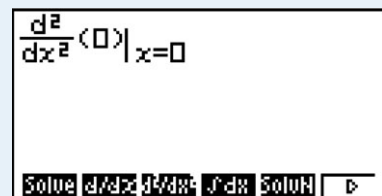
Press **VAR** | **F4** GRPH | **F1** Y then type 1(0) to enter Y1(0).

Similarly enter Y1(3).

The stationary points are at (0, 0) and (3, -27).



Press **OPTN** **F4** CALC | **F3** $\frac{d^2}{dx^2}$ to enter the second derivative template.

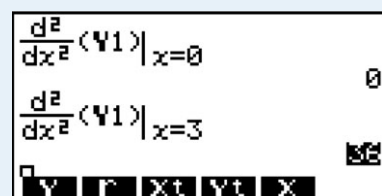


Enter Y1 in the template as the function using the same procedure to enter the Y.

Enter the value of x as 0.

Repeat for the second derivative when $x = 3$.

In this case we are not certain what the nature of the stationary point is at (0, 0) but the point (3, -27) is a minimum because $f''(x) > 0$.

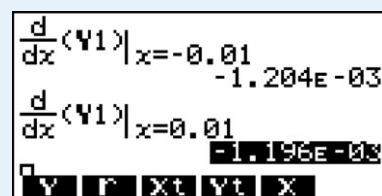


Evaluate $f'(x)$ either side of $x = 0$, in this case using $x = -0.01$ and $x = 0.01$.

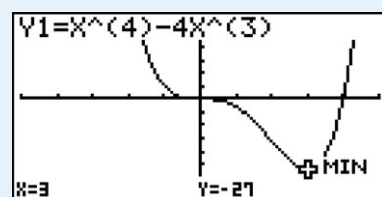
To evaluate the derivatives follow the same steps as before, but use **F2** $\frac{d}{dx}$.

The gradient is negative either side of the stationary point.

Hence (0, 0) is a negative point of inflection.



The graph on the right illustrates the curve, the minimum at (3, -27) and the point of inflection at (0, 0).



3 Integral calculus

The calculator can find the values of definite integrals either on a calculator page or graphically.

The calculator method is quicker, but the graphical method is clearer and shows discontinuities, negative areas and other anomalies that can arise.

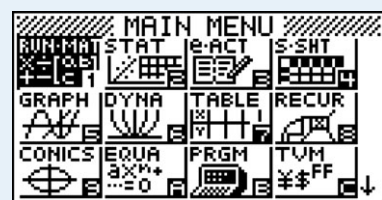
3.1 Finding the value of an indefinite integral

Example 26

Evaluate $\int_2^8 \left(x - \frac{3}{\sqrt{x}} \right) dx$

Press **MENU**. You will see the dialog box as shown on the right.

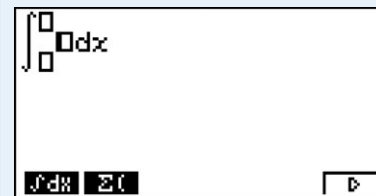
Choose 1: RUN·MAT and press **EXE**.



▶ Continued on next page

Press **OPTN** **F4** CALC | **F6** $\frac{\square}{\square}$ | **F1** $\int dx$ to enter the second derivative template.

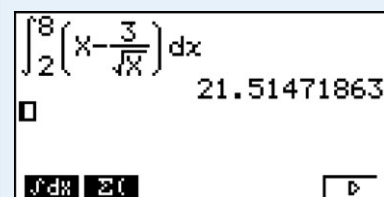
In this example you will also use templates to enter the rational function and the square root.



Enter the upper and lower limits, the function and x in the template.

Press **EXE**.

The value of the integral is 21.5 (to 3 sf).



3.2 Finding the area under a curve

Example 27

$$y = 3x^2 - 5$$

Find the area bounded by the curve, the x -axis and the lines $x = -1$ and $x = 1$.

Press **MENU**. You will see the dialog box as shown on the right.

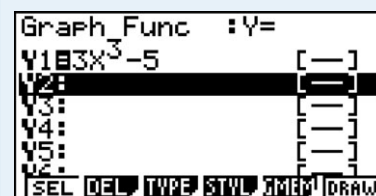


Choose 5: GRAPH and press **EXE**.

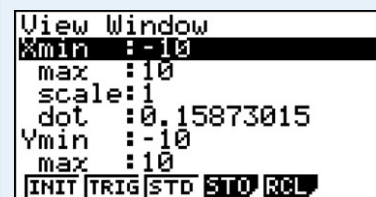
Press **OPTN** **F2** CALC.



Enter the function $y = 3x^2 - 5$ and press **EXE**.

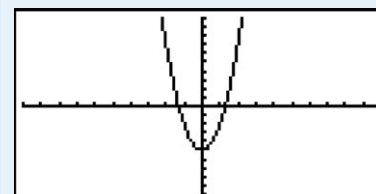


Press **SHIFT** **F3** V-Window and choose **F3** STD for the default axes which are $-10 \leq x \leq 10$ and $-10 \leq y \leq 10$.



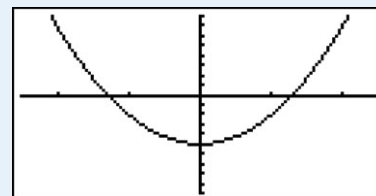
Press **EXE** and **F6** DRAW.

The graph of $y = 3x^2 - 5$ is displayed with the default axes.



► Continued on next page

Change the axes to make the curve fit the screen better.

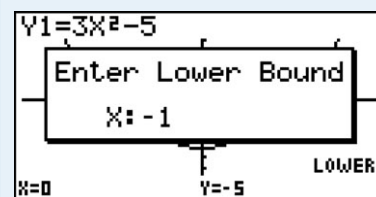


Press **F5** G-Solv | **F6** \uparrow | **F3** $\int dx$.

The calculator prompts you to enter the lower limit for the integral.

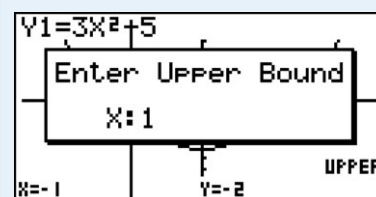
Type -1 and press **EXE**.

Be sure to use the **(-)** key.



The calculator prompts you to enter the upper limit for the integral.

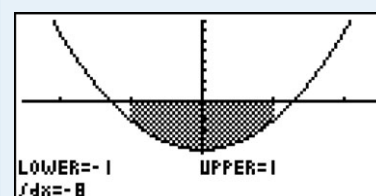
Type 1 and press **EXE**.



The area found is shaded and the value of the integral (-8) is shown on the screen.

The required area is 8.

Note: since the area lies below the x-axis in this case, the integral is negative.



4 Vectors

Scalar product

4.1 Calculating a scalar product

Example 28

Evaluate the scalar products:

$$\mathbf{a} \begin{pmatrix} 1 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 4 \end{pmatrix} \quad \mathbf{b} \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$$

Press **MENU**. You will see the dialog box as shown on the right.

Choose 1: RUN·MAT and press **EXE**.



Press **OPTN** **F1** LIST | **F6** \rightarrow | **F6** \rightarrow | **F1** Sum.



▶ Continued on next page

Enter the vectors as lists using curly brackets { }. Separate the terms of the vectors using commas.

Multiply the two vector lists together.

Close the bracket and press **ENTER**.

$$\begin{pmatrix} 1 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} -3 \\ 4 \end{pmatrix} = 9$$

Press **OPTN** **F1** LIST | **F6** **▶** | **F6** **▶** | **F1** Sum.

Enter the vectors as lists using curly brackets { }. Separate the terms of the vectors using commas.

Multiply the two vector lists together.

Close the bracket and press **ENTER**.

$$\begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} = -3$$

4.2 Calculating the angle between two vectors

The angle θ between two vectors \vec{a} and \vec{b} , can be calculated using the formula

$$\theta = \arccos\left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}\right)$$

Example 29

Calculate the angle between $2\vec{i} + 3\vec{j}$ and $3\vec{i} - \vec{j}$.

Press **SHIFT** SET UP.

Select either Deg or Rad using **F1** or **F2**.

Press **EXE**.

Press **SHIFT** (\cos^{-1}) $\frac{a}{b/c}$ to select the fraction template.

Press **OPTN** **F1** LIST | **F6** **▶** | **F6** **▶** | **F1** Sum.

▶ Continued on next page

Enter the vectors as lists using curly brackets { }. Separate the terms of the vectors using commas.

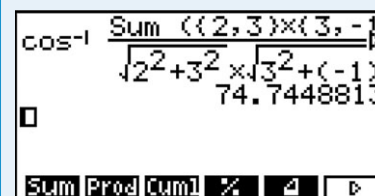
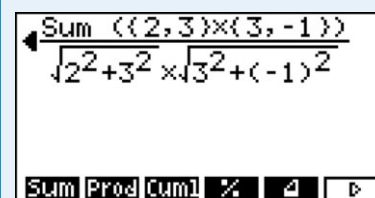
Multiply the two vector lists together.

To calculate the magnitudes of the vectors use the formula

$$|a\vec{i} + b\vec{j}| = \sqrt{a^2 + b^2}$$

Press **EXE**.

The angle between $2\vec{i} + 3\vec{j}$ and $3\vec{i} - \vec{j}$ is 74.7° .



5 Statistics and probability

You can use your GDC to draw charts to represent data and to calculate basic statistics such as mean, median, etc. Before you do this you need to enter the data in a list.

Entering data

There are two ways of entering data: as a list or as a frequency table.

5.1 Entering lists of data

Example 30

Enter the data in the list: 1, 1, 3, 9, 2.

Press **menu**. You will see the dialog box as shown on the right.

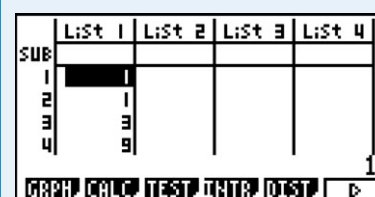
Choose 2: STAT and press **EXE**.



Type the numbers in the first column (List 1).

Press **EXE** after each number to move down to the next cell.

List 1 will be used later when you want to make a chart or to do some calculations with this data. You can use columns from List 1 to List 26 to enter the list.



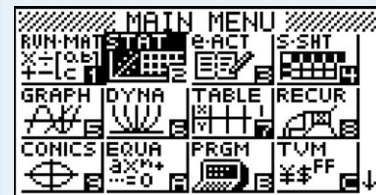
5.2 Entering data from a frequency table

Example 31

Enter the data in the table:

Number	1	2	3	4	5
Frequency	3	4	6	5	2

Press **menu**. You will see the dialog box as shown on the right.
Choose the 2: STAT and press **EXE**.



Type the numbers in the first column (List 1) and the frequencies in the second column (List 2).

Press **EXE** after each number to move down to the next cell. Press **▶** to move to the next column.

List 1 and List 2 will be used later when you want to make a chart or to do some calculations with this data. You can use columns from List 1 to List 26 to enter the lists.

SUB	List 1	List 2	List 3	List 4
1	1	3		
2	2	4		
3	3	5		
4	4	6		

Drawing charts

Charts can be drawn from a list or from a frequency table.

5.3 Drawing a frequency histogram from a list

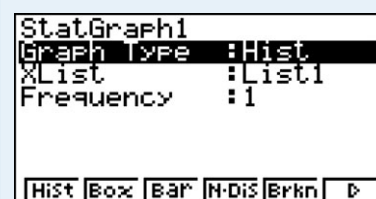
Example 32

Draw a frequency histogram for this data:
1, 1, 3, 9, 2.

Enter the data in List 1 (see Example 5).

Press **F1** GRPH and **F6** SET.

By default the graph type is Scatter Diagram (Scat). Change this to Histogram (Hist) and leave XList as List 1 and Frequency as 1.



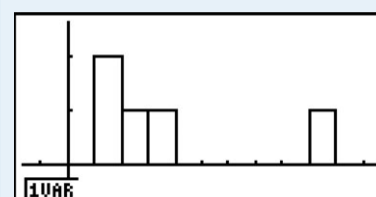
Press **EXIT** and press **F1** to select GPH1.

Change the start value for the histogram to 0 and the width of the bars to 1.

Press **EXE**.



A histogram, scaled for the data in the list is displayed.

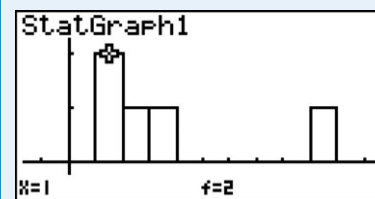


▶ Continued on next page

Press **SHIFT** **F1** (Trace).

Use the **▶** key to move to each of the bars and display their value and frequency.

You should now see a frequency histogram for the data in the list 1, 1, 3, 9, 2.



5.4 Drawing a frequency histogram from a frequency table

Example 33

Draw a frequency histogram for this data:

Number	1	2	3	4	5
Frequency	3	4	6	5	2

Enter the data in L1 and L2 (see Example 6).

Press **F1** GRPH and **F6** SET.

By default the graph type is Scatter Diagram (Scat). Change this to Histogram (Hist). Leave XList as List 1 and set the Frequency as List 2.

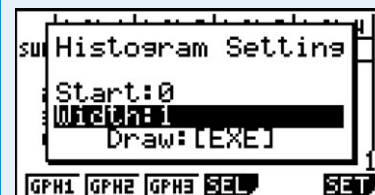
You should see this display.



Press **EXIT** and press **F1** to select GPH1.

Change the start value for the histogram to 0 and the width of the bars to 1.

Press **EXE**.

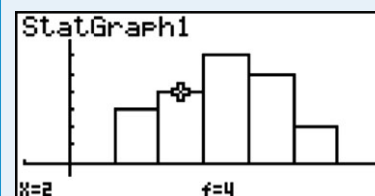


A histogram, scaled for the data in the list is displayed.



Press **SHIFT** **F1** (Trace).

Use the **▶** key to move to each of the bars and display their value and frequency.



5.5 Drawing a box and whisker diagram from a list

Example 34

Draw a box and whisker diagram for this data:
1, 1, 3, 9, 2.

Enter the data in List 1 (see Example 5).

Press **F1** GRPH and **F6** SET.

By default the graph type is Scatter Diagram (Scat). Change this to MedBox (Box) and leave XList as List 1, the Frequency as 1 and Outliers off.

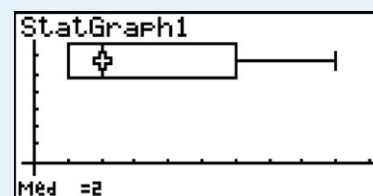
```
StatGraph1
Graph Type : MedBox
XList      : List1
Frequency  : 1
Outliers   : Off
|GPH1 |GPH2 |GPH3
```

Press **EXIT** and press **F1** to select GPH1.



Press **SHIFT** **F1** (Trace).

Use the **▶** key to move the cursor over the plot to see the quartiles, Q1 and Q3, the median and the maximum and minimum values.



5.6 Drawing a box and whisker diagram from a frequency table

Example 35

Draw a box and whisker diagram for this data:

Number	1	2	3	4	5
Frequency	3	4	6	5	2

Enter the data in L1 and L2 (see Example 6).

Press **F1** GRPH and **F6** SET.

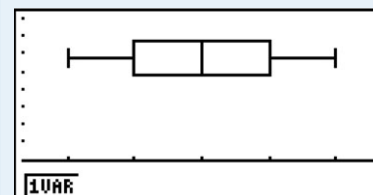
By default the graph type is Scatter Diagram (Scat). Change this to MedBox (Box). Leave XList as List 1, set the Frequency as List 2 and leave Outliers off.

```
StatGraph1
Graph Type : MedBox
XList      : List1
Frequency  : List2
Outliers   : Off
|GPH1 |GPH2 |GPH3
```

You should see this display.

```
StatGraph1
Graph Type : MedBox
XList      : List1
Frequency  : List2
Outliers   : Off
|GPH1 |GPH2 |GPH3
```

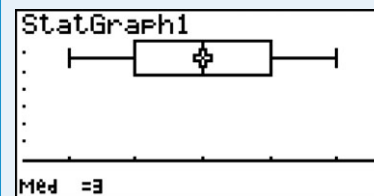
Press **EXIT** and press **F1** to select GPH1.



▶ Continued on next page

Press **SHIFT** **F1** (Trace).

Use the **▶** key to move the cursor over the plot to see the quartiles, Q1 and Q3, the median and the maximum and minimum values.



Calculating statistics

You can calculate statistics such as mean, median, etc. from a list, or from a frequency table.

5.7 Calculating statistics from a list

Example 36

Calculate the summary statistics for this data: 1, 1, 3, 9, 2

Enter the data in List 1 (see Example 5).

Press **F2** **CALC** and **F6** **SET**.

Use the default values for 1 variable statistics which are XList as List 1 and Freq as 1.

```
1Var XList :List1
1Var Freq  :1
2Var XList :List1
2Var YList :List2
2Var Freq  :1
LIST
```

Press **EXIT** and press **F1** to select 1 VAR.

	List 1	List 2	List 3	List 4
SUB				
1	1			
2	1			
3	3			
4	9			

1VAR 2VAR REG SET

The information shown will not fit onto a single screen.

You can scroll up and down to see it all.

The statistics calculated for the data are:

mean	\bar{x}
sum	$\sum x$
sum of squares	$\sum x^2$
population standard deviation	σx
sample standard deviation	Sx
number	n
minimum value	$\min X$
lower quartile	Q_1
median	Med
upper quartile	Q_3
maximum value	$\max X$
mode	Mod
number of data mode items	Mod:n
data mode frequency	Mod:F

```
1-Variable
x̄ = 3.2
Σx = 16
Σx² = 96
σx = 2.9933259
sx = 3.3466401
n = 5
```

```
1-Variable
n = 5
minX = 1
Q1 = 1
Med = 2
Q3 = 3
maxX = 9
```

```
1-Variable
Med = 2
Q3 = 3
maxX = 9
Mod = 1
Mod:n = 1
Mod:F = 2
```

5.8 Calculating statistics from a frequency table

Example 37

Calculate the summary statistics for this data:

Number	1	2	3	4	5
Frequency	3	4	6	5	2

Enter the data in L1 and L2 (see Example 6).

Press **F2** CALC and **F6** SET.

Leave the 1 variable statistics default value for XList as List 1 and change Freq to List 2.

```
1Var XList :List1
2Var Freq :List2
20 Select List No.
20 List[1~26]: 2
1 LIST
```

You should see this display.

```
1Var XList :List1
1Var Freq :List2
20Var XList :List1
20Var YList :List2
20Var Freq :1
1 LIST
```

Press **EXIT** and press **F1** to select 1 VAR.

```
List 1 List 2 List 3 List 4
SUB
1 1 3
2 2 4
3 3 6
4 4 5
1VAR 2VAR REG SET
```

The information shown will not fit onto a single screen.

You can scroll up and down to see it all.

The statistics calculated for the data are:

mean	\bar{x}
sum	$\sum x$
sum of squares	$\sum x^2$
population standard deviation	σx
sample standard deviation	Sx
number	n
minimum value	minX
lower quartile	Q_1
median	Med
upper quartile	Q_3
maximum value	maxX
mode	Mod
number of data mode items	Mod:n
data mode frequency	Mod:F

```
1-Variable
x̄ =2.95
Σx =59
Σx² =203
σx =1.20312094
sx =1.23437604
n =20 ↓
```

```
1-Variable
n =20 ↑
minX =1
Q1 =2
Med =3
Q3 =4
maxX =5 ↓
```

```
1-Variable
Med =3 ↑
Q3 =4
maxX =5
Mod =3
Mod:n=1
Mod:F=6
```


5.9 Calculating the interquartile range

Example 38

Calculate interquartile range for this data:

Number	1	2	3	4	5
Frequency	3	4	6	5	2

The interquartile range is the difference between the upper and lower quartiles ($Q_3 - Q_1$).

Press **menu**. You will see the dialog box as shown on the right.
Choose 1: RUN·MAT and press **EXE**.



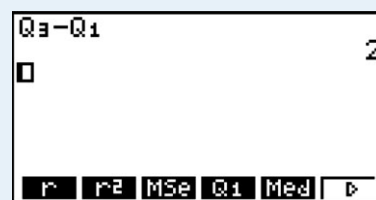
First calculate the summary statistics for this data (see Example 12).

(**Note:** The values of the summary statistics are stored after 1-Variable Statistics have been calculated and remain stored until the next time they are calculated.)

Press **VAR** | **F3** STAT | **F3** GRPH | **F6** **F6** | **F1** Q_3 **-** **F6** **F6** | **F4** Q_1 **EXE**

The calculator now displays the result:

Interquartile range = $Q_3 - Q_1 = 2$



5.10 Using statistics

The calculator stores the values you calculate in One-Variable Statistics so that you can access them in other calculations. These values are stored until you do another One-Variable Statistics calculation.

Example 39

Calculate $\bar{x} + \sigma x$ for this data:

Number	1	2	3	4	5
Frequency	3	4	6	5	2

Press **menu**. You will see the dialog box as shown on the right.
Choose 1: RUN·MAT and press **EXE**.



First calculate the summary statistics for this data (see Example 12).

(**Note:** The values of the summary statistics are stored after 1-Variable Statistics have been calculated and remain stored until the next time they are calculated.)

Press **VAR** | **F1** X | **F2** \bar{x} + **F5** σx **EXE**

The calculator now displays the result:

$\bar{x} + \sigma x = 4.15$ (to 3sf)



Calculating binomial probabilities

5.11 The use of nCr

Example 40

Find the value of $\binom{8}{3}$ (or ${}_8C_3$).

Press **menu**. You will see the dialog box as shown on the right.
Choose 1: RUN·MAT and press **EXE**.



Press **OPTN** **F6** **▶** | **F3** PROB to enter probability menu.
Press **8** **F3** nCr **3** **EXE**.



Example 41

List the values of $\binom{4}{r}$ for $r = 0, 1, 2, 3, 4$.

Press **menu**. You will see the dialog box as shown on the right.
Choose 7: TABLE and press **EXE**.



Press **OPTN** **F4** PROB to enter probability menu.
Press **4** **F3** nCr **X,θ,T** **EXE**.

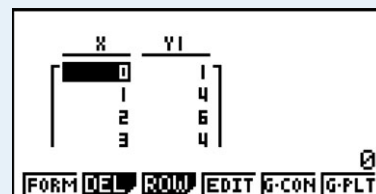


Press **F6** TABL.

The table shows that

$$\binom{4}{0}=1, \binom{4}{0}=1, \binom{4}{1}=4, \binom{4}{2}=6, \binom{4}{3}=4 \text{ and } \binom{4}{4}=1$$

You may need to reset the start value and step values for the table using **F5** SET.



5.12 Calculating binomial probabilities

Example 42

X is a discrete random variable and $X \sim \text{Bin}(9, 0.75)$.

Calculate $P(X = 5)$

$$P(X = 5) = \binom{9}{5} 0.75^5 0.25^4$$

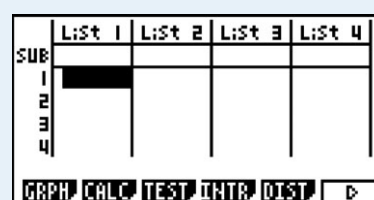
The calculator can find this value directly.

Press **menu**. You will see the dialog box as shown on the right.

Choose 2: STAT and press **EXE**.

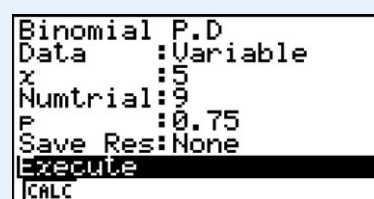


Press **F5** DIST | **F5** BINM | **F1** Bpd.

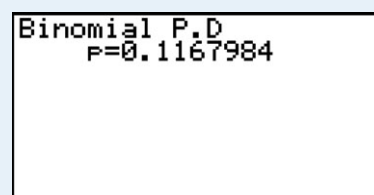


Enter the x -value 5, the number of trials, 9 and the probability 0.75.

Press **EXE**.



The calculator shows that $P(X = 5) = 0.117$ (to 3 sf).



Example 43

X is a discrete random variable and $X \sim \text{Bin}(7, 0.3)$.

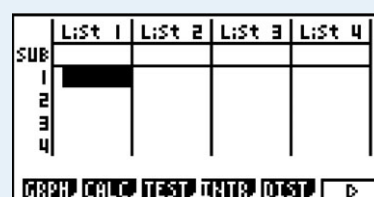
Calculate the probabilities that X takes the values $\{0, 1, 2, 3, 4, 5, 6, 7\}$.

Press **menu**. You will see the dialog box as shown on the right.

Choose 2: STAT and press **EXE**.



Press **F5** DIST | **F5** BINM | **F1** Bpd.



► Continued on next page

Enter the values 0,1,2,3,4,5,6,7 in List1. Press **EXE** after each number.
Press **F5** DIST | **F5** BINM | **F1** Bpd.

	List 1	List 2	List 3	List 4
SUB				
1	0			
2	1			
3	2			
4	3			
				0

GRAPH CALC TEST INTD DIST D

Press **F1** List.
Choose List1.
Enter the number of trials, 7 and the probability 0.3.
Press **EXE**.

Binomial P.D
Data :List
List :List1
Numtrial:7
P :0.3
Save Res:None
Execute
ICALC

The calculator displays each of the probabilities.
To see the remaining values scroll the screen down.

Note: the figures on the left refer to the positions of the values in the list, not the values themselves.

Binomial P.D
1 0.0823
2 0.2471
3 0.3176
4 0.2268
5 0.0972
0.0823543

Example 44

X is a discrete random variable and $X \sim \text{Bin}(20, 0.45)$.

Calculate

- the probability that X is less than or equal to 10.
- the probability that X lies between 5 and 15 inclusive.
- the probability that X is greater than 11.

Press **menu**. You will see the dialog box as shown on the right.
Choose 2: STAT and press **EXE**.

MAIN MENU			
RUN-MAT	STAT	E-ACT	S-SHT
X=0	X=1	X=2	X=3
GRAPH	DYNA	TABLE	RECUR
CONICS	EQUA	PRGM	TVM
3X+1	3X+2	3X+3	3X+4

Press **F5** DIST | **F5** BINM | **F2** Bcd.

You are given the lower bound probability so you have to calculate other probabilities using this.

	List 1	List 2	List 3	List 4
SUB				
1				
2				
3				
4				

GRAPH CALC TEST INTD DIST D

- Enter the x -value 5, the number of trials, 9 and the probability 0.75.
Press **EXE**.

Binomial C.D
Data :Variable
x :10
Numtrial:20
P :0.45
Save Res:None
Execute
ICALC

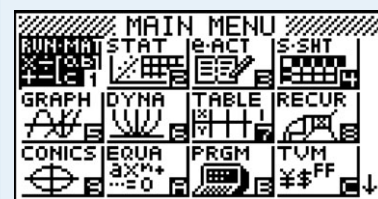
$$P(X \leq 10) = 0.751 \text{ (to 3 sf)}$$

Binomial C.D
P=0.75071064

► Continued on next page

- b** When using the lower bound probability to calculate other probabilities, it is easier to do this without the wizard.

Press **MENU**. Choose 1: RUN·MAT and press **EXE**.



$$P(5 \leq X \leq 15) = P(X \leq 15) - P(X \leq 4)$$

Press **OPTN** **F5** STAT | **F3** DIST | **F5** BINM to enter the binomial menu.

Enter binominalCD(15, 20, 0.45) – binominalCD(4, 20, 0.45)

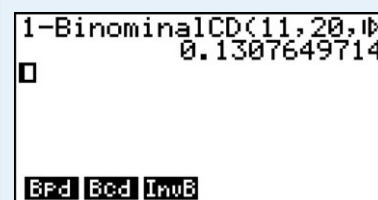
$$P(5 \leq X \leq 15) = 0.980 \text{ (to 3 sf)}$$



c $P(X > 11) = 1 - P(X \leq 11)$

Enter 1 – binominalCD (11, 20, 0.45)

$$P(X > 11) = 0.131 \text{ (to 3 sf)}$$



Calculating normal probabilities

5.13 Calculating normal probabilities from X-values

Example 45

A random variable X is normally distributed with a mean of 195 and a standard deviation of 20 or $x \sim N(195, 20)^2$. Calculate

- the probability that X is less than 190.
- the probability that X is greater than 194.
- the probability that X lies between 187 and 196.

Press **menu** and choose 2: STAT and press **EXE**.



Press **F5** DIST **F1** NORM **F2** Ncd to use the Normal Cumulative Distribution function.



Press **F2** Var.

The value 1E99 is the largest value that can be entered in the GDC and is used in the place of ∞ . It stands for 1×10^{99} (–1E99 is the smallest value and is used in the place of $-\infty$). To enter the E, you need to press **EXP**.



► Continued on next page

This dialogue box is used to calculate normal probabilities.

You should enter the values, Lower Bound, Upper Bound, σ and μ , in order.

```
Normal C.D
Data :Variable
Lower :0
Upper :0
σ :1
μ :0
Save Res:None
List Var ↓
```

a $P(x < 190)$

Enter Lower Bound as $-1E99$, Upper Bound as 190, σ as 20 and μ as 195.

Press **EXE**.

```
Normal C.D
Data :Variable
Lower :-1E+99
Upper :190
σ :20
μ :195
Save Res:None
[None] List ↓
```

$P(X < 190) = 0.401$ (to 3 sf)

```
Normal C.D
P =0.40129367
z:Low=-5E+97
z:UP =-0.25
```

b $P(x > 194)$

Press **EXIT** to return to the entry screen.

Enter Lower Bound as 194, Upper Bound as $1E99$, σ as 20 and μ as 195.

```
Normal C.D
Data :Variable
Lower :194
Upper :1E+99
σ :20
μ :195
Save Res:None
[None] List ↓
```

$P(X > 194) = 0.520$ (to 3 sf)

```
Normal C.D
P =0.5199388
z:Low=-0.05
z:UP =5E+97
```

c $P(187 < x < 196)$

Press **EXIT** to return to the entry screen.

Enter Lower Bound as 187, Upper Bound as 196, μ as 195 and σ as 20.

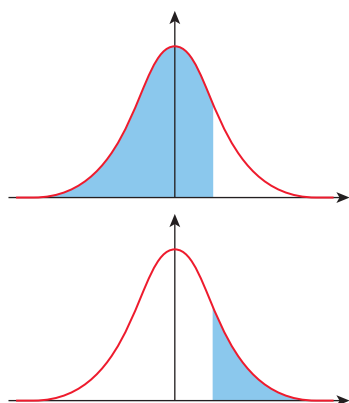
```
Normal C.D
Data :Variable
Lower :187
Upper :196
σ :20
μ :195
Save Res:None
[None] List ↓
```

$P(187 < X < 196) = 0.175$ (to 3 sf)

```
Normal C.D
P =0.17536054
z:Low=-0.4
z:UP =0.05
```


5.14 Calculating X-values from normal probabilities

In some problems you are given probabilities and have to calculate the associated values of X . To do this, use the Inverse Normal function.



When using the Inverse Normal function, make sure you find the probability on the correct side of the normal curve. If the area is the lower tail, that is it is in the form $P(X < x)$, use 'Left' tail (see Example 26).

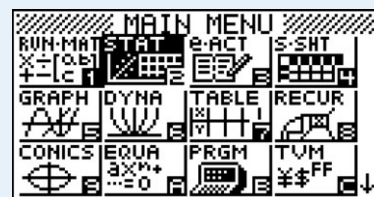
If you are given the upper tail $P(X > x)$, you should use 'Right' tail (see Example 27).

Example 46

A random variable X is normally distributed with a mean of 75 and a standard deviation of 12 or $X \sim N(75, 12^2)$. If $P(X < x) = 0.4$, find the value of x .

You are given a lower-tail probability so you should choose 'Left'.

Press **menu** and choose 2: STAT and press **EXE**.

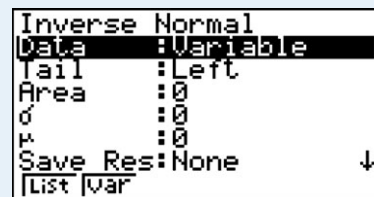


Press **F5** DIST **F1** NORM **F3** InvN to use the Inverse Normal function.



Press **F2** Var.

You should enter the values: Area (probability), σ and μ , in order.

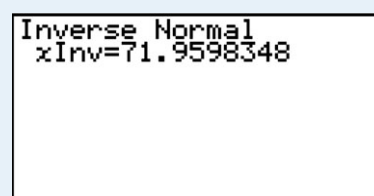


Enter Tail as Left (**F1**), Area as 0.4, σ as 12 and μ as 75.

Press **EXE**.



So if $P(X < x) = 0.4$ then $x = 72.0$ (to 3 sf).



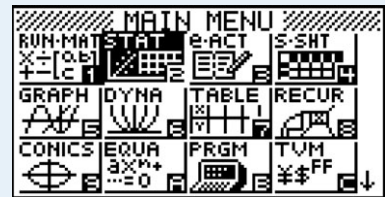
Example 47

A random variable X is normally distributed with a mean of 75 and a standard deviation of 12 or $X \sim N(75, 12^2)$.

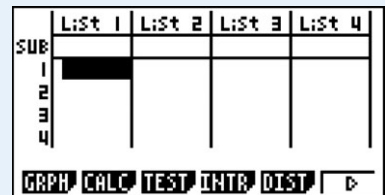
If $P(X > x) = 0.2$, find the value of x .

You are given an upper-tail probability so you should choose 'Right'.

Press **menu** and choose 2: STAT and press **EXE**.

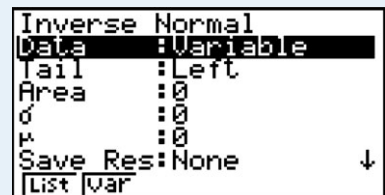


Press **F5** DIST **F1** NORM **F3** InvN to use the Inverse Normal function.



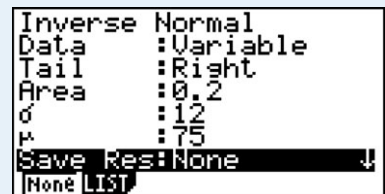
Press **F2** Var.

You should enter the values: Area (probability), σ and μ , in order.

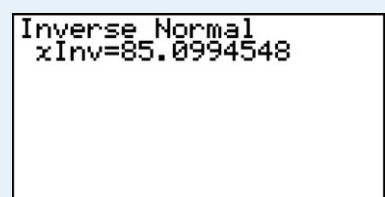


Enter Tail as Right (**F2**), Area as 0.2, σ as 12 and μ as 75.

Press **EXE**.



So if $P(X > x) = 0.2$ then $x = 85.1$ (to 3 sf).



Scatter diagrams, linear regression and the correlation coefficient

5.15 Scatter diagrams

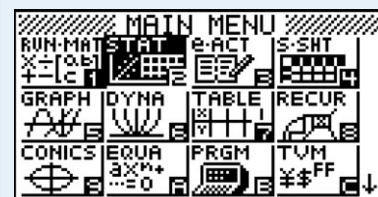
Example 48

Consider this data that is approximately connected by a linear function.

x	1.0	2.1	2.4	3.7	5.0
y	4.0	5.6	9.8	10.6	14.7

- Find the equation of the least squares regression line of y on x .
- Find Pearson's product-moment correlation coefficient.
- Use the equation to predict the value of y when $x = 3.0$.

Press **menu**. You will see the dialog box as shown on the right.
Choose the 2: STAT and press **EXE**.



Type the x -values in the first column (List 1) and the y -values in the second column (List 2).

Press **EXE** after each number to move down to the next cell. Press **▶** to move to the next column.

	List 1	List 2	List 3	List 4
SUB				
1				
2	2.1	5.6		
3	2.4	9.8		
4	3.7	10.6		

GRAPH CALC TEST INTD DIST

Press **F1** GRPH and **F6** SET.

	List 1	List 2	List 3	List 4
SUB				
1				
2	2.1	5.6		
3	2.4	9.8		
4	3.7	10.6		

GP1 GP2 GP3 SEL SET

Select Graph Type and choose **F1** Scatter.

Press **EXE**.

StatGraph1	
Graph Type	: Scatter
XList	: List1
YList	: List2
Frequency	: 1
Mark Type	: *

Scat XY NFF Pie

Press **F1** GPH1.

The automatic scales do not always give the best display of the box and whisker diagram. You cannot use V-Window to change the default values, but you can zoom in or out.



▶ Continued on next page

Press **F1** CALC.

Press **F2** x and **F1** $ax+b$.

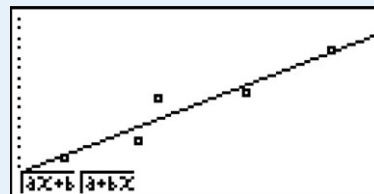
You will see the coefficients of the equation of the least squares regression line and the value r of the correlation coefficient.

The equation is $y = 2.63x + 1.48$ (to 3 sf).

The value of r is 0.955 (to 3 sf).

```
LinearReg(ax+b)
a =2.62819974
b =1.47591271
r =0.95474098
r^2=0.91153034
MSe=2.1301133
y=ax+b
COPY DRAW
```

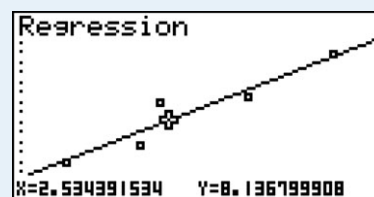
Press **F6** DRAW and you will see the least squares regression line and the data points that you plotted previously.



Press **SHIFT** **F1** TRACE and use the **▶** **◀** keys to move the trace along the line.

The cursor moves along the regression line.

Press **▲** to move between the data points.



It is not possible to move the trace point to an exact value, so get as close to $x = 3$ as you can.

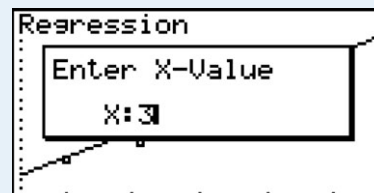
From the graph, you have found that y is approximately 9.5 when $x = 3.0$.



Press **SHIFT** **F5** G-SLV.

Press **F1** Y-CAL.

Press **3** **EXE**.



The cursor moves to exactly 3.

When $x = 3.0$, an estimate of the value of y is 9.36, from the graph.

